Discovering Opinion Intervals from Conflicts in Signed Graphs

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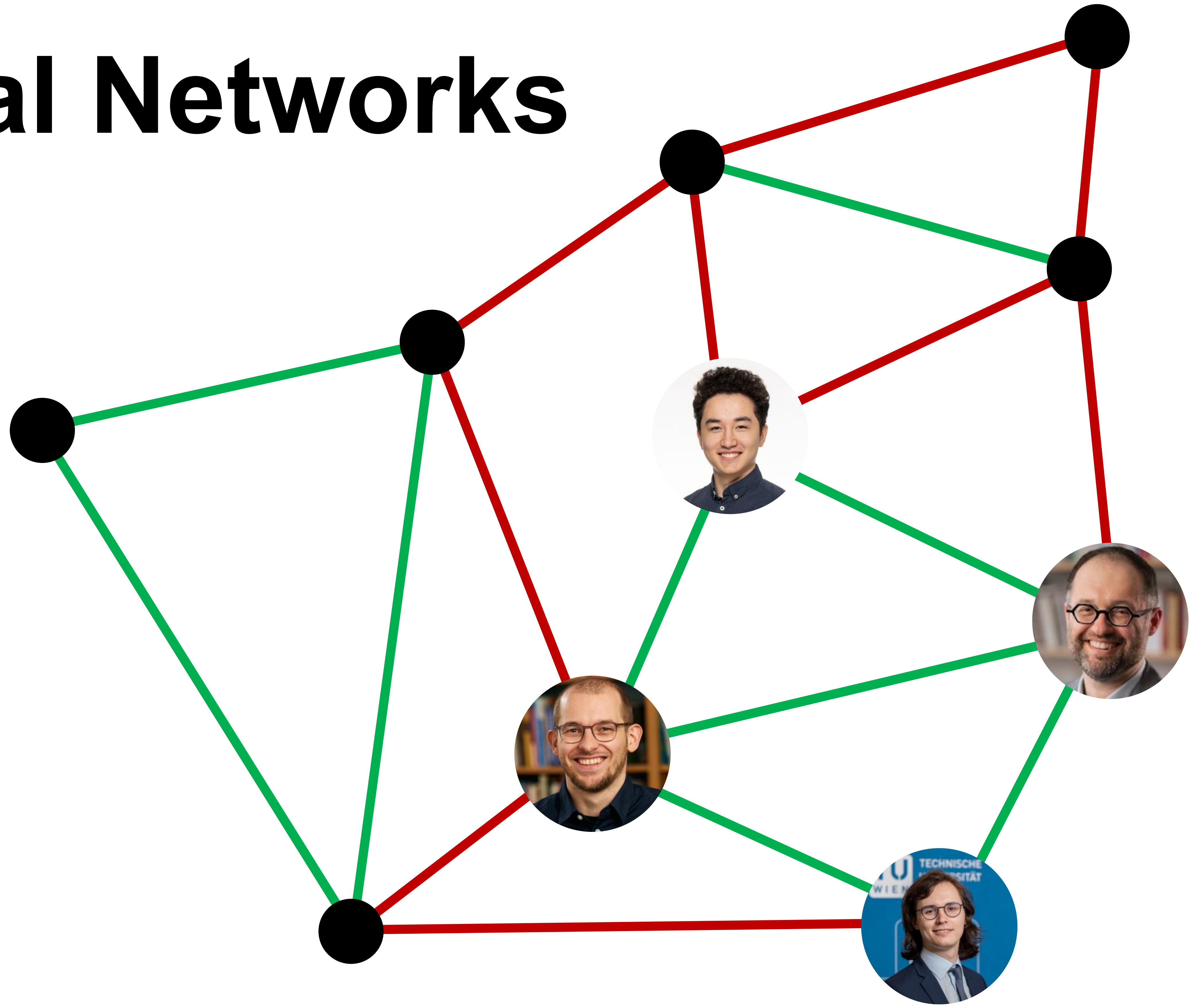
Interactions in Social Networks

San Diego Sightseeing Preferences

- We observe a social network modelled as a **signed** graph



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- Each edge has a **sign** + or – for
  - Positive + interactions or
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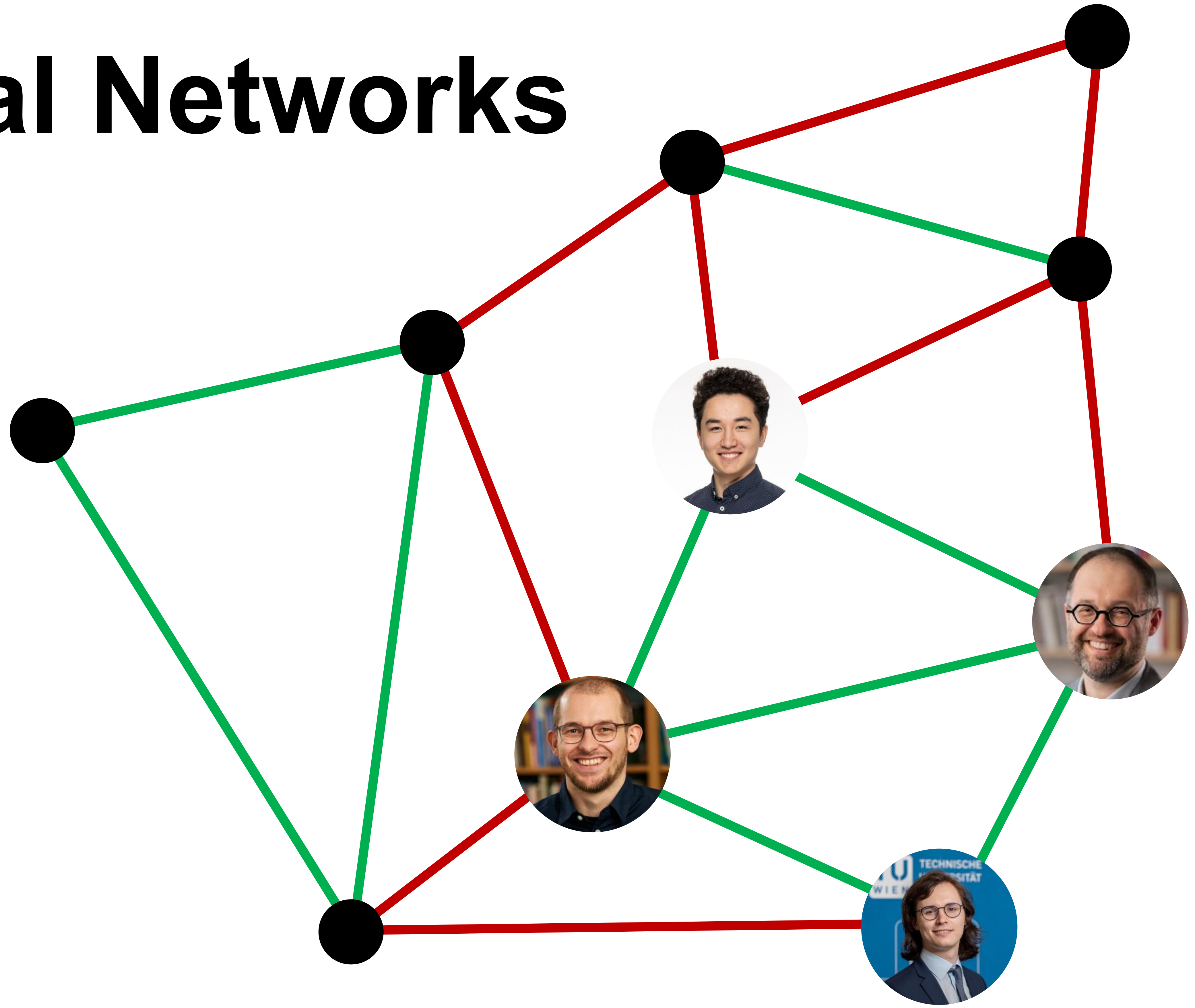

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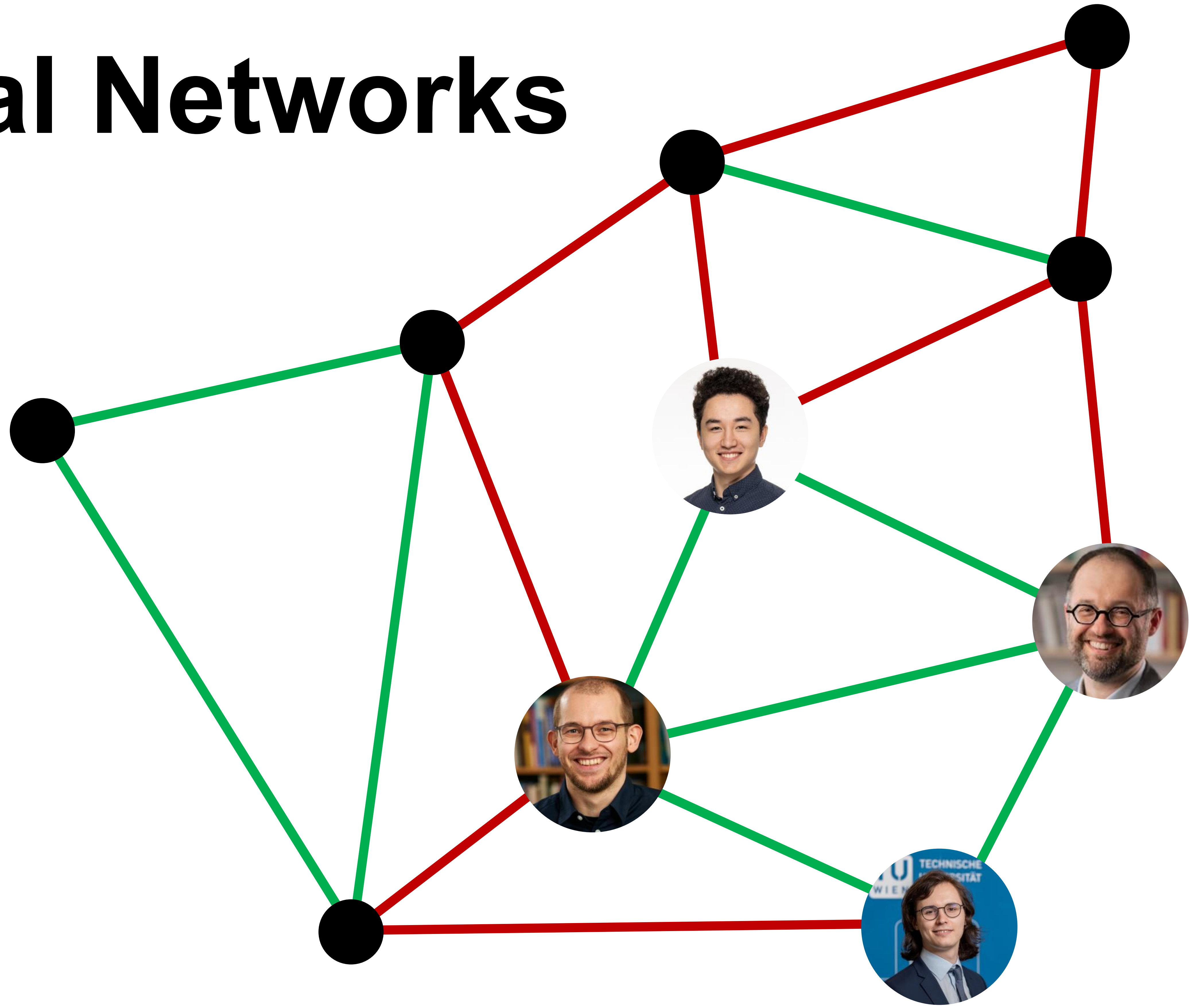
- **Goal:** Discover individuals' opinions



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CORRELATION CLUSTERING:

Assign each vertex $v \in V$ a cluster label $\ell_v \in \mathbb{N}$ to maximize the number of

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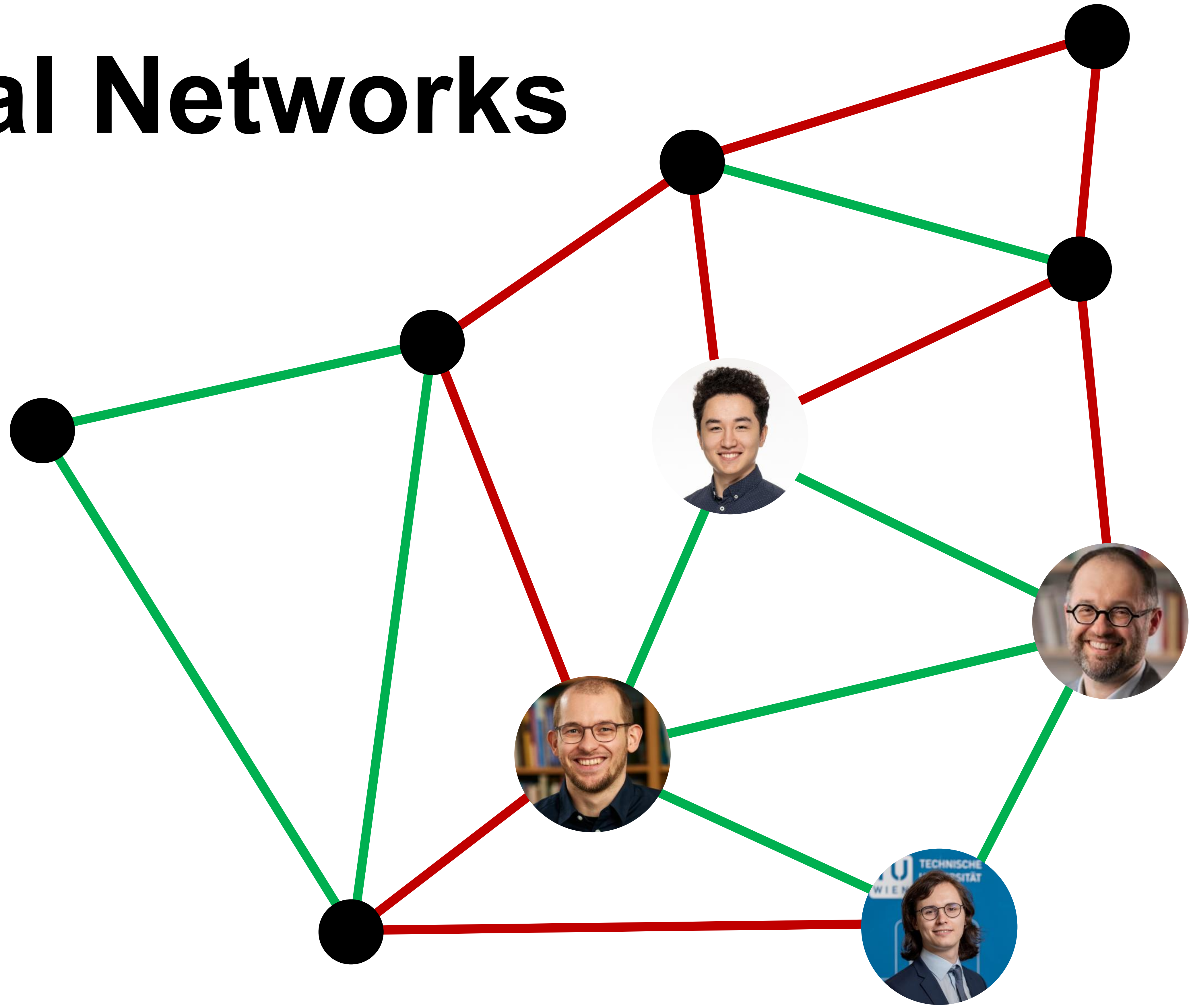

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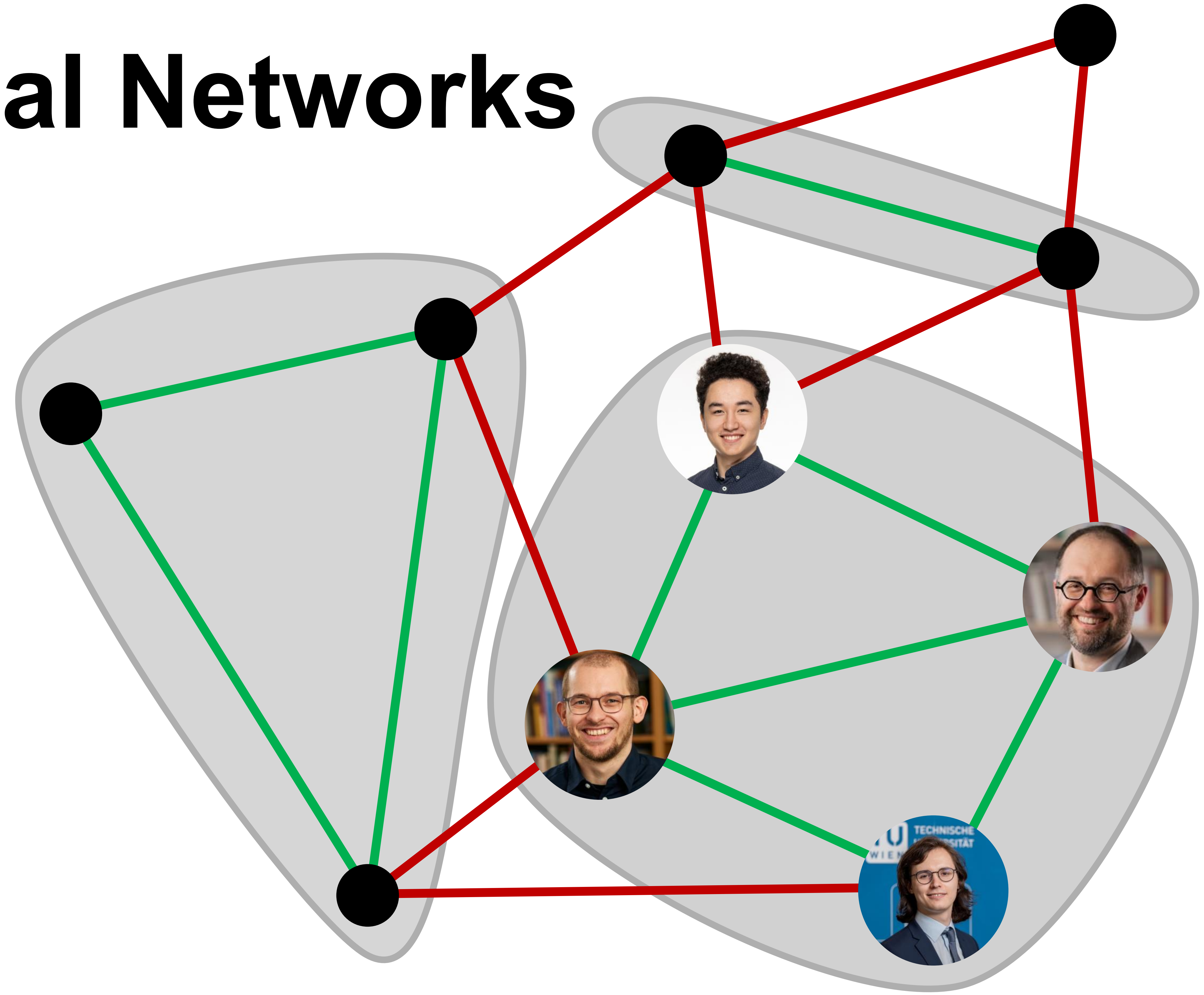

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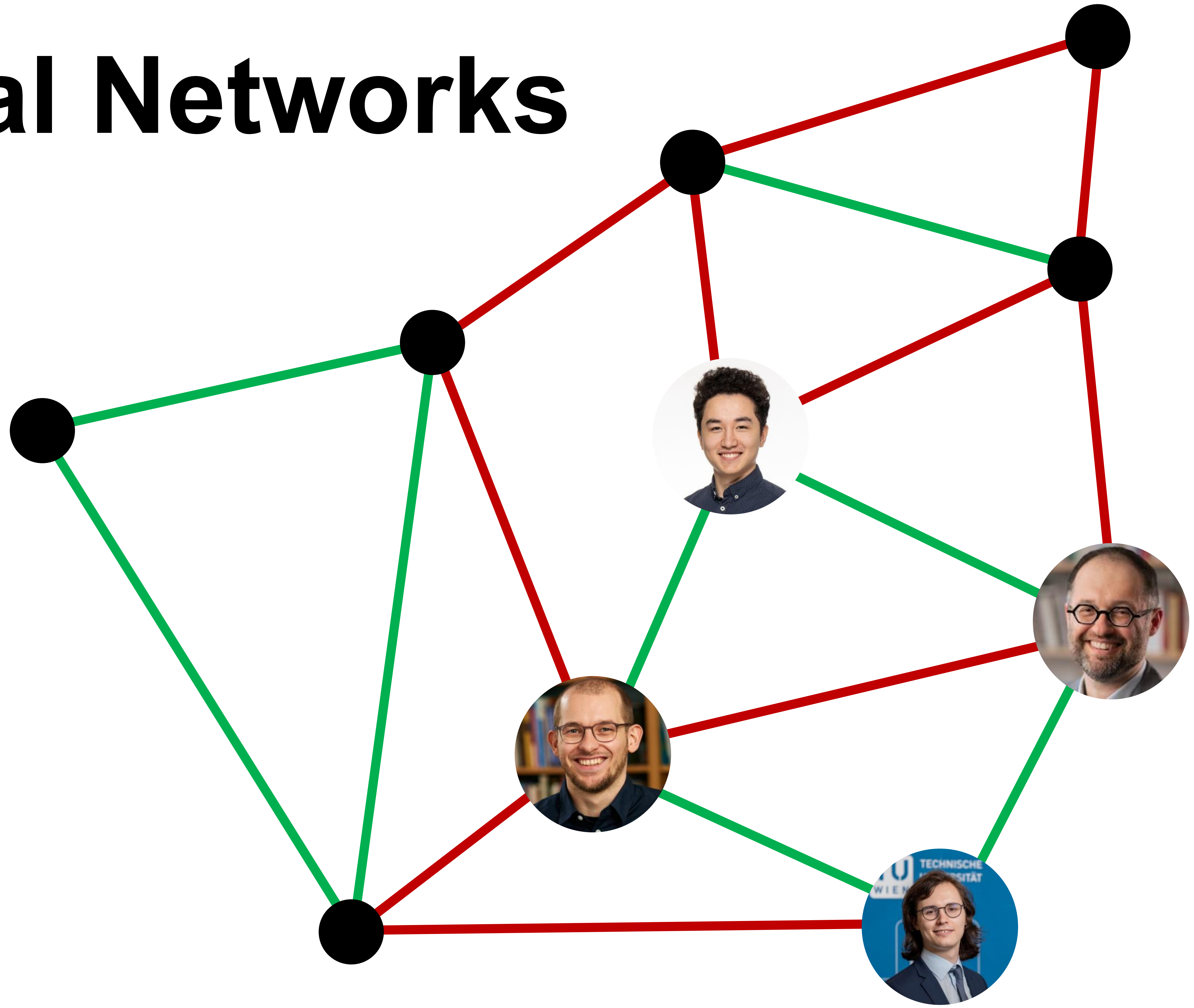

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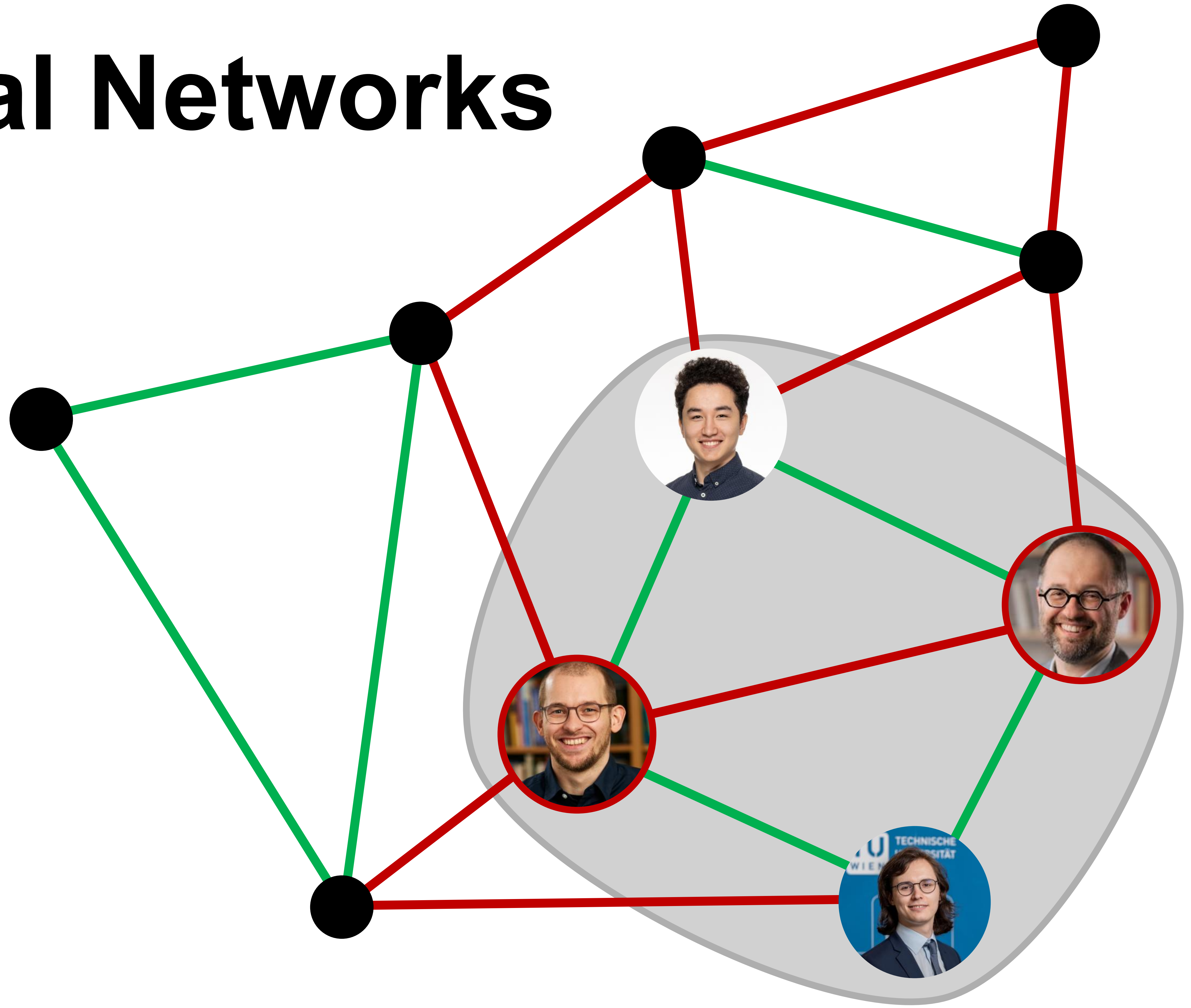
Problem: Disjoint clusters cannot model complex node interactions



Interactions in Social Networks

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Best Interval Approximation:

Assign each vertex an interval $\{I_v : v \in V\}$ with $I_v \subset \mathbb{R}$ to maximize the number of

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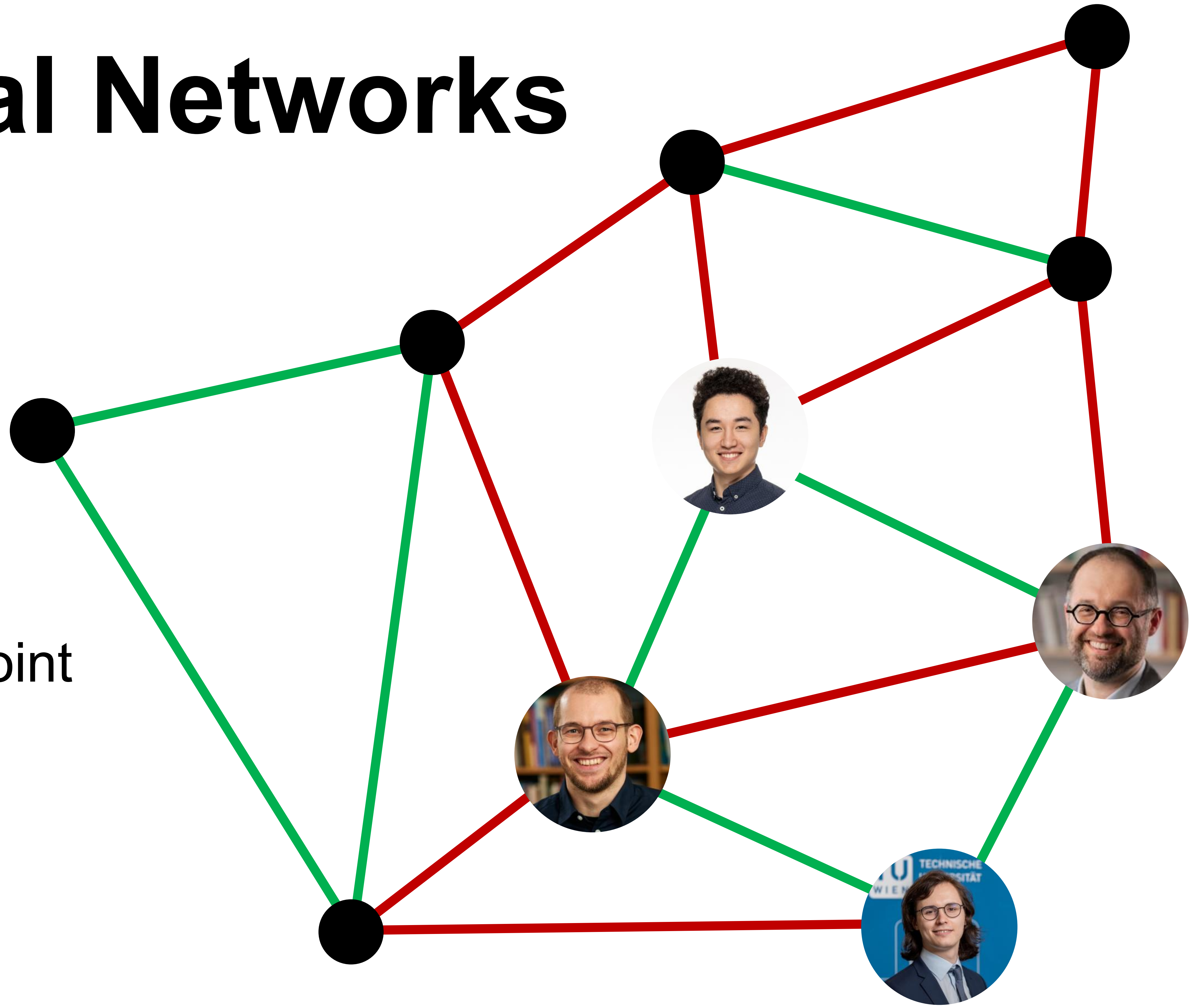

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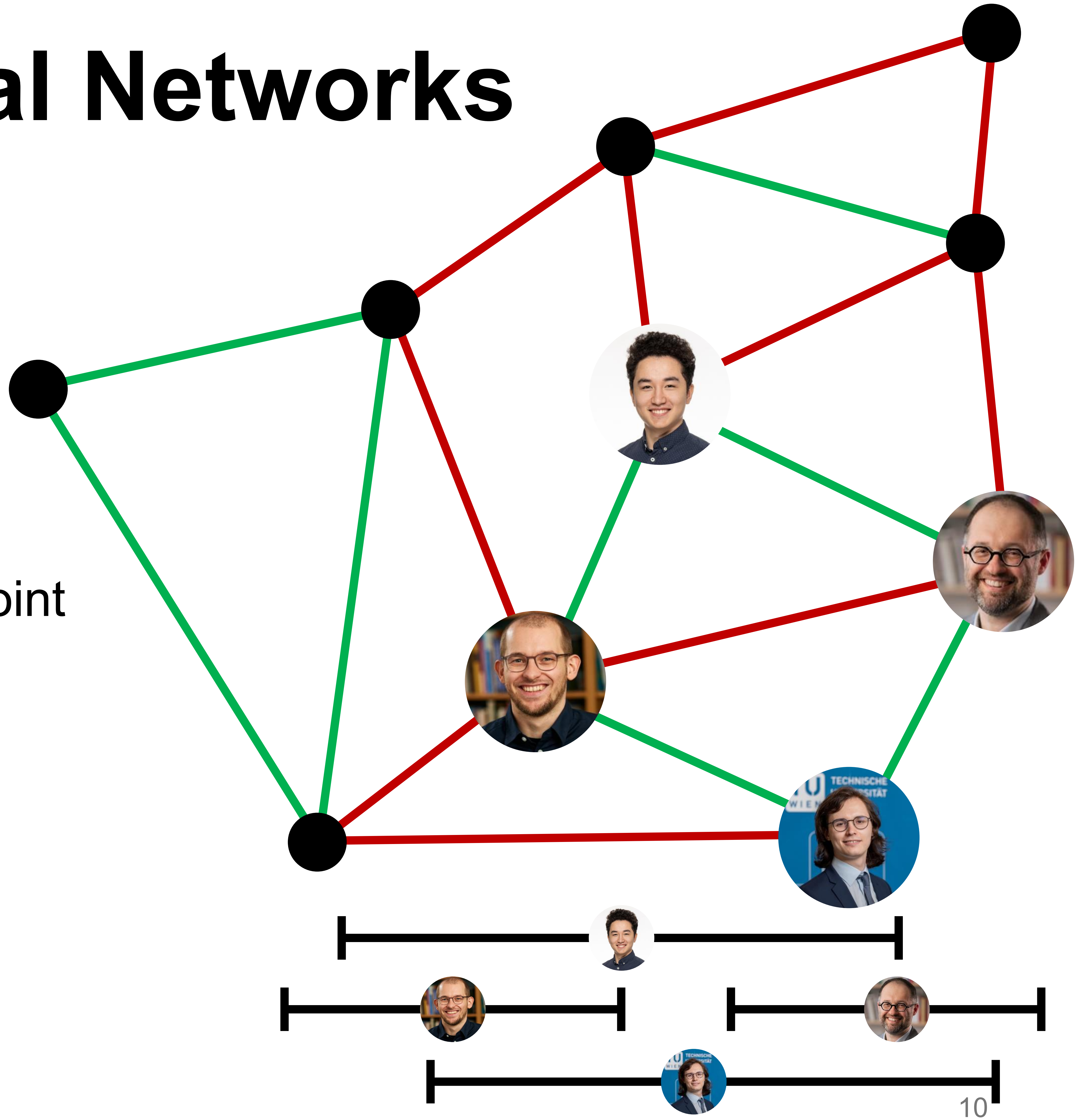

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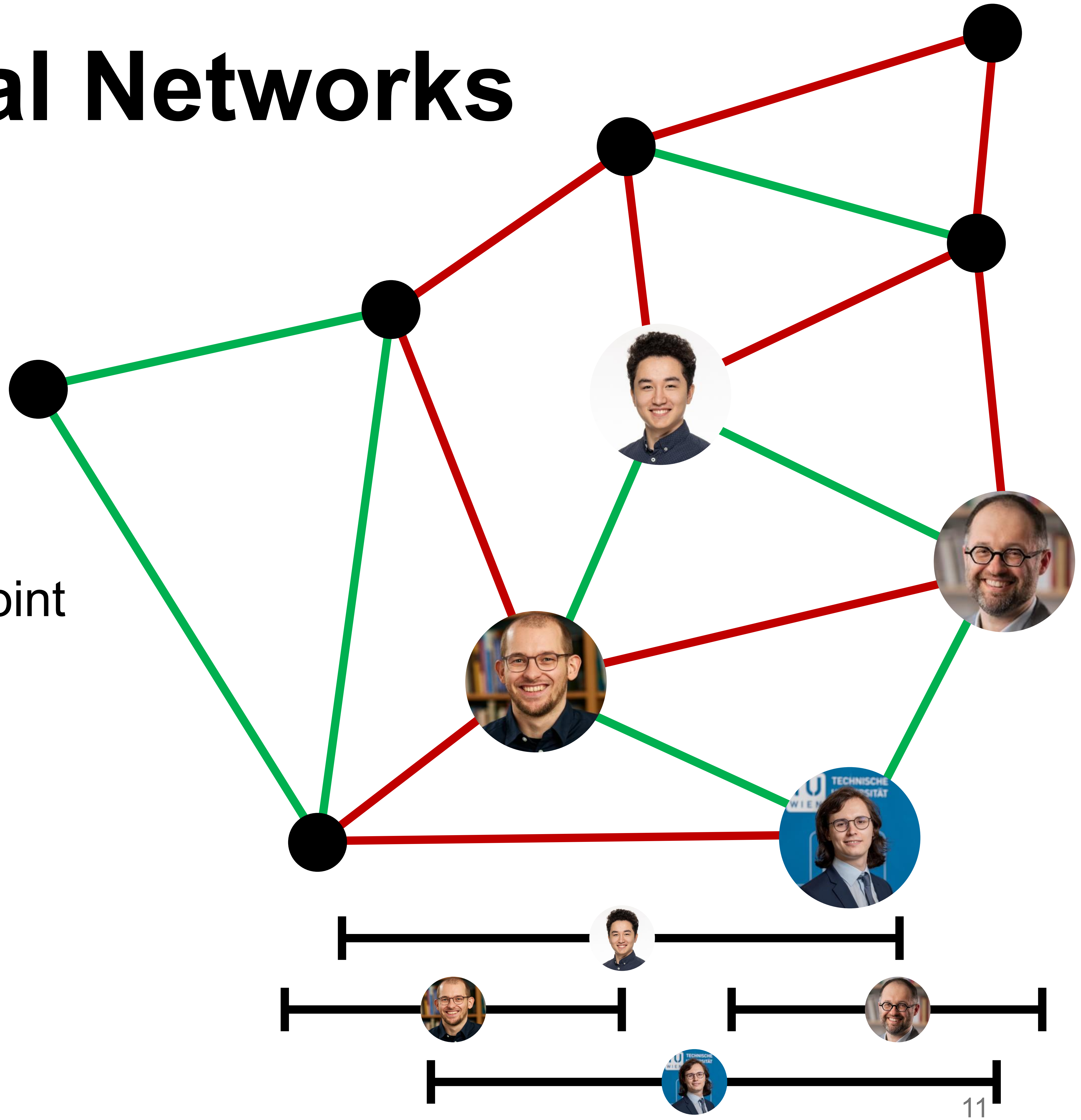

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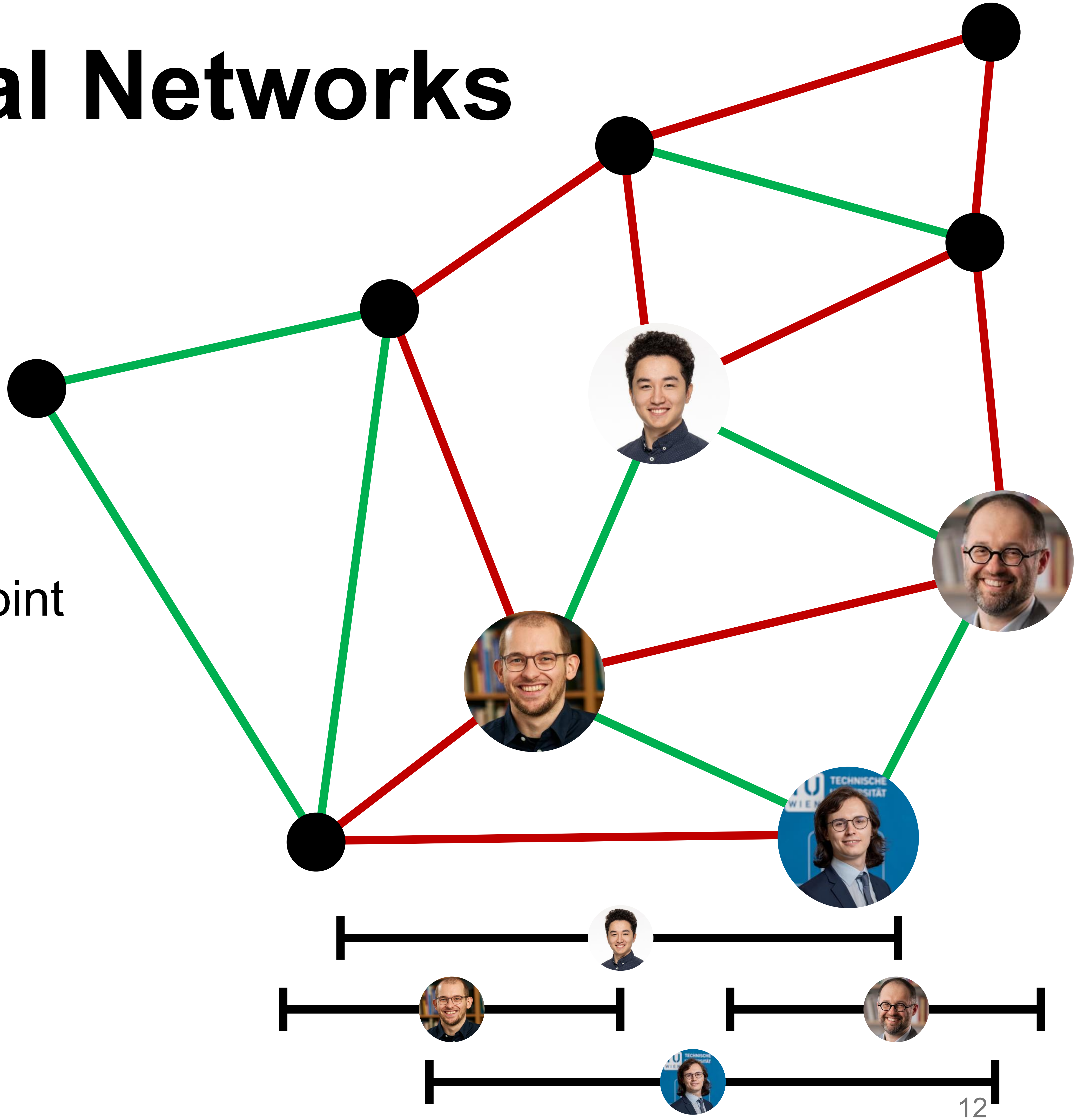

- This is more expressive than CORRELATION CLUSTERING
- The model captures the **tolerance** an individual has for other opinions

Problem Analysis

NP-Hardness

Theorem

BEST INTERVAL APPROXIMATION is NP-hard. This follows via a reduction from ACYCLIC DIGRAPH PARTITION.

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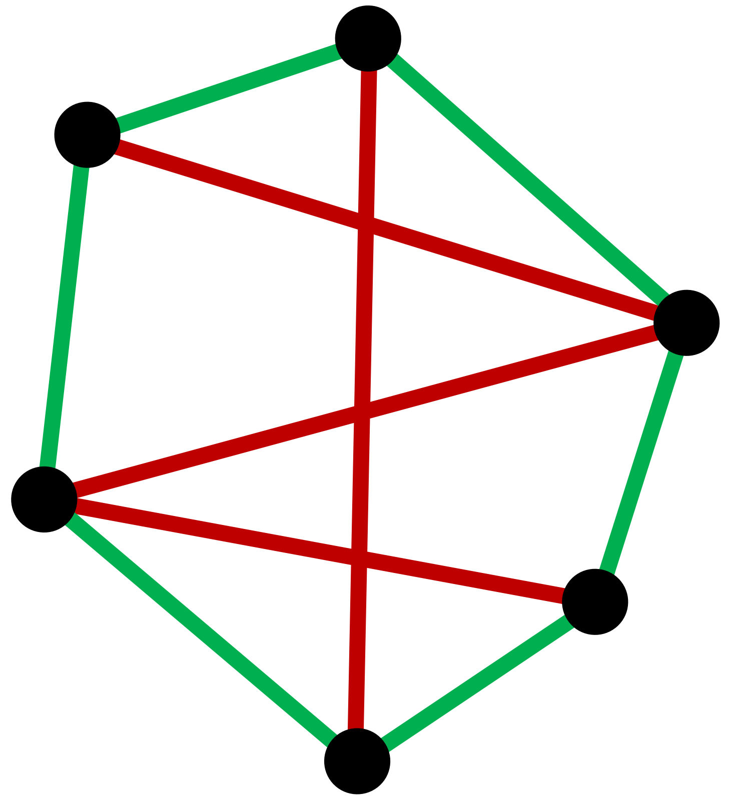

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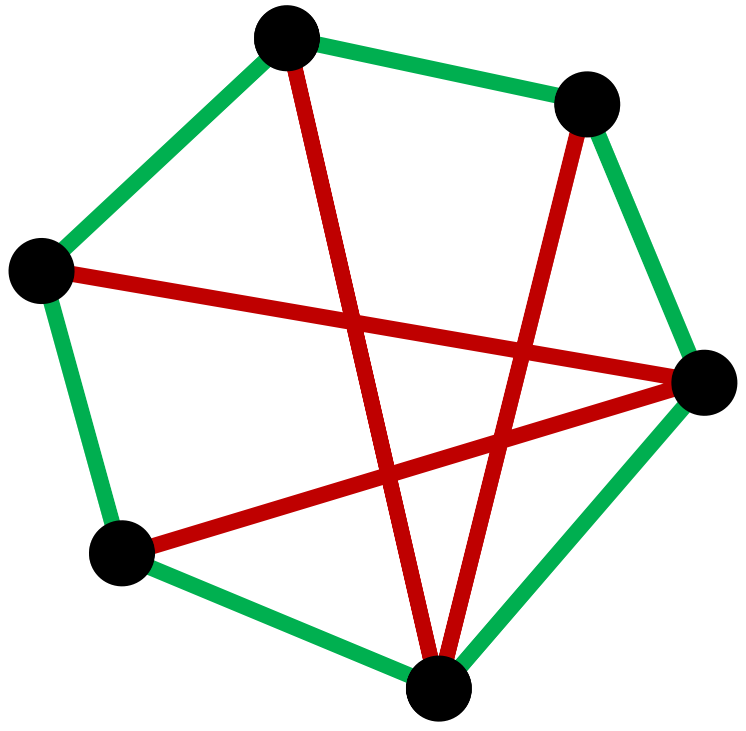


Algorithms

PTAS in restricted setting

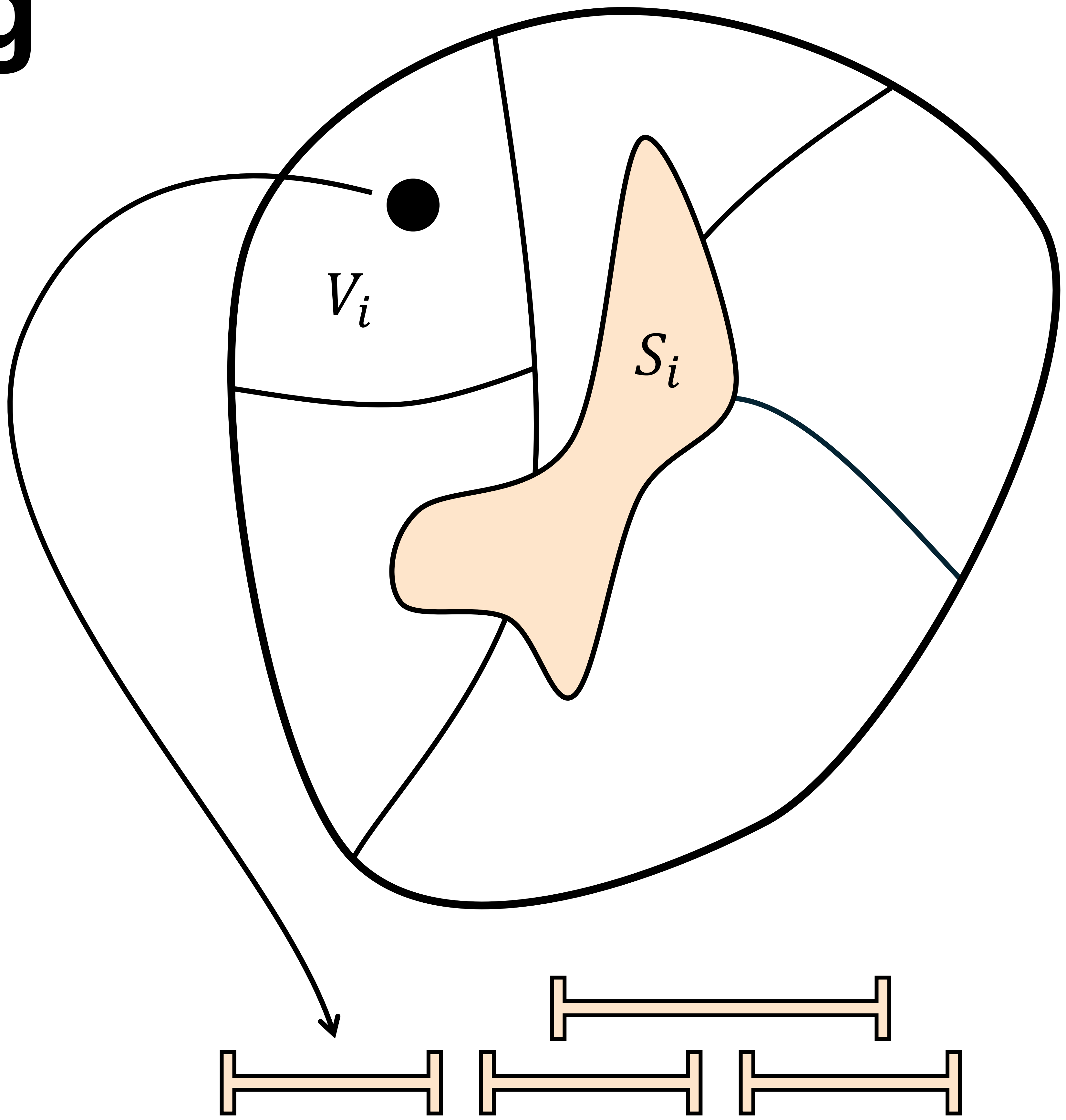
For complete signed graphs and fixed interval configurations find a $(1+\epsilon)$-approximation:

1. Partition V into sets $V_1, \dots, V_m$
2. For each subset V_i:

   Sample S_i with replacement from $V \setminus V_i$

   Solve S_i optimally

   Assign V_i greedily
3. Combine solutions across all V_i



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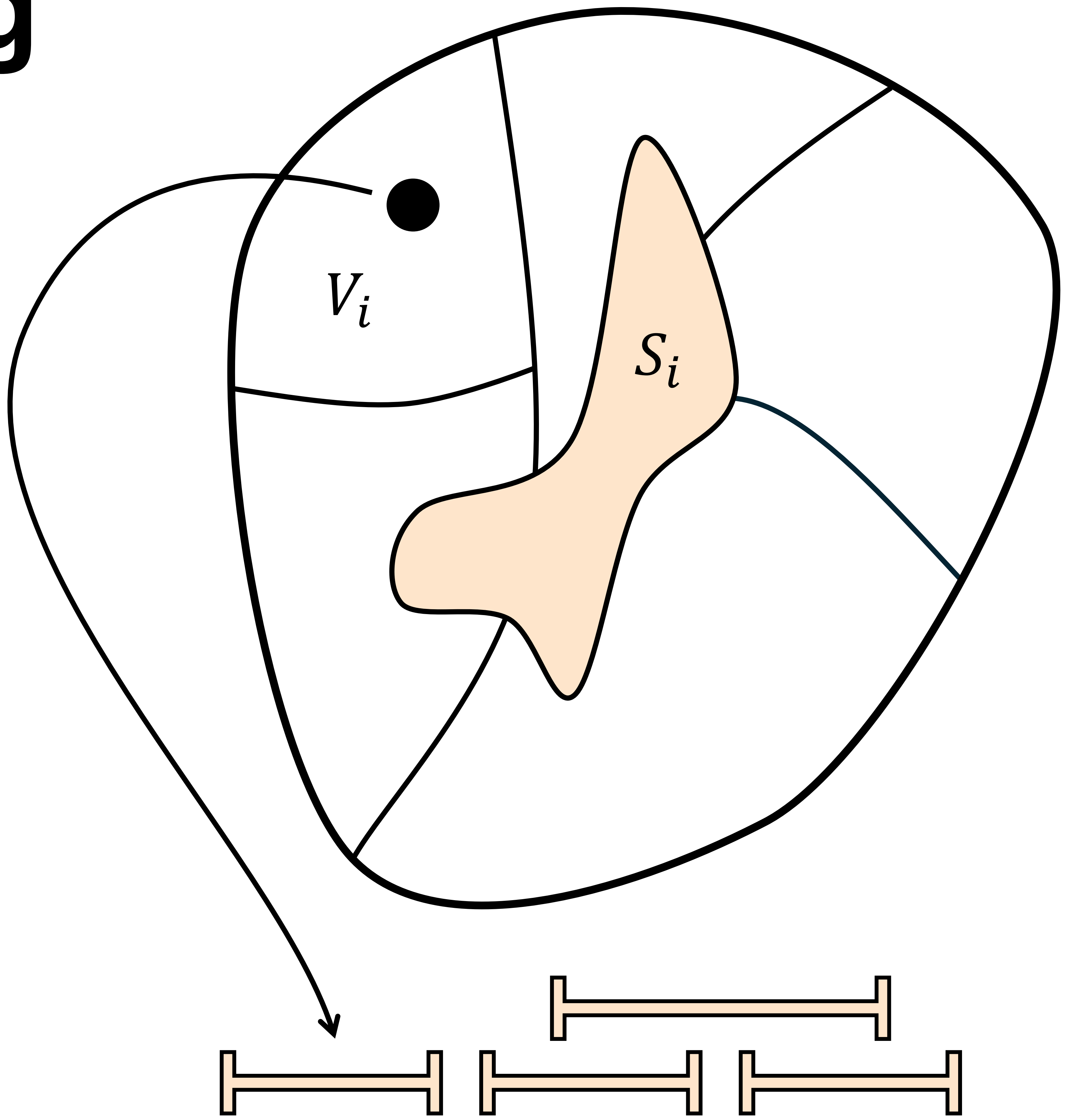


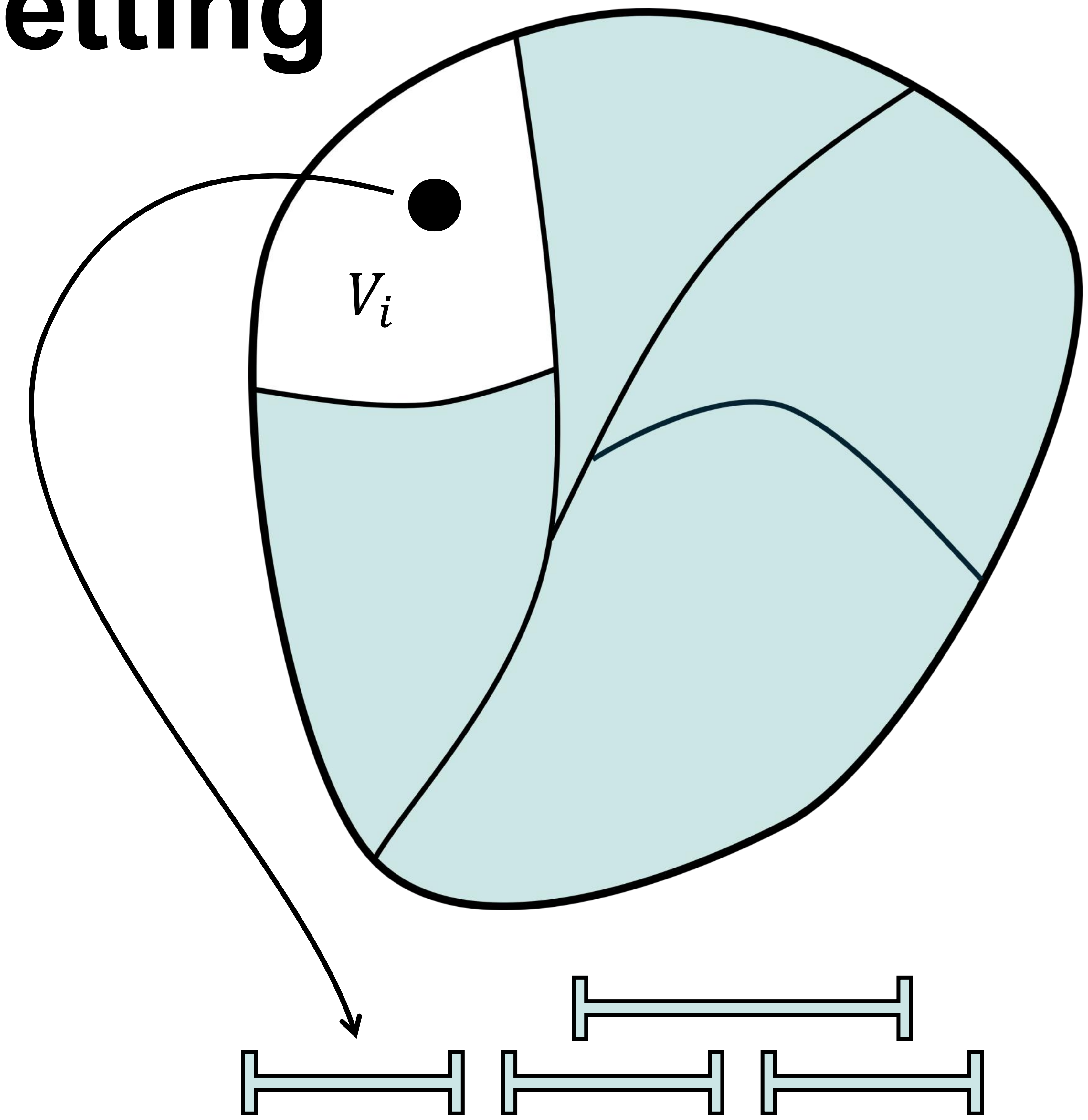
Heuristics in restricted setting

For ~~complete~~ signed graphs and fixed interval configurations find a heuristic solution:

1. Partition V into sets $V_1, \dots, V_m$
2. For each subset V_i:

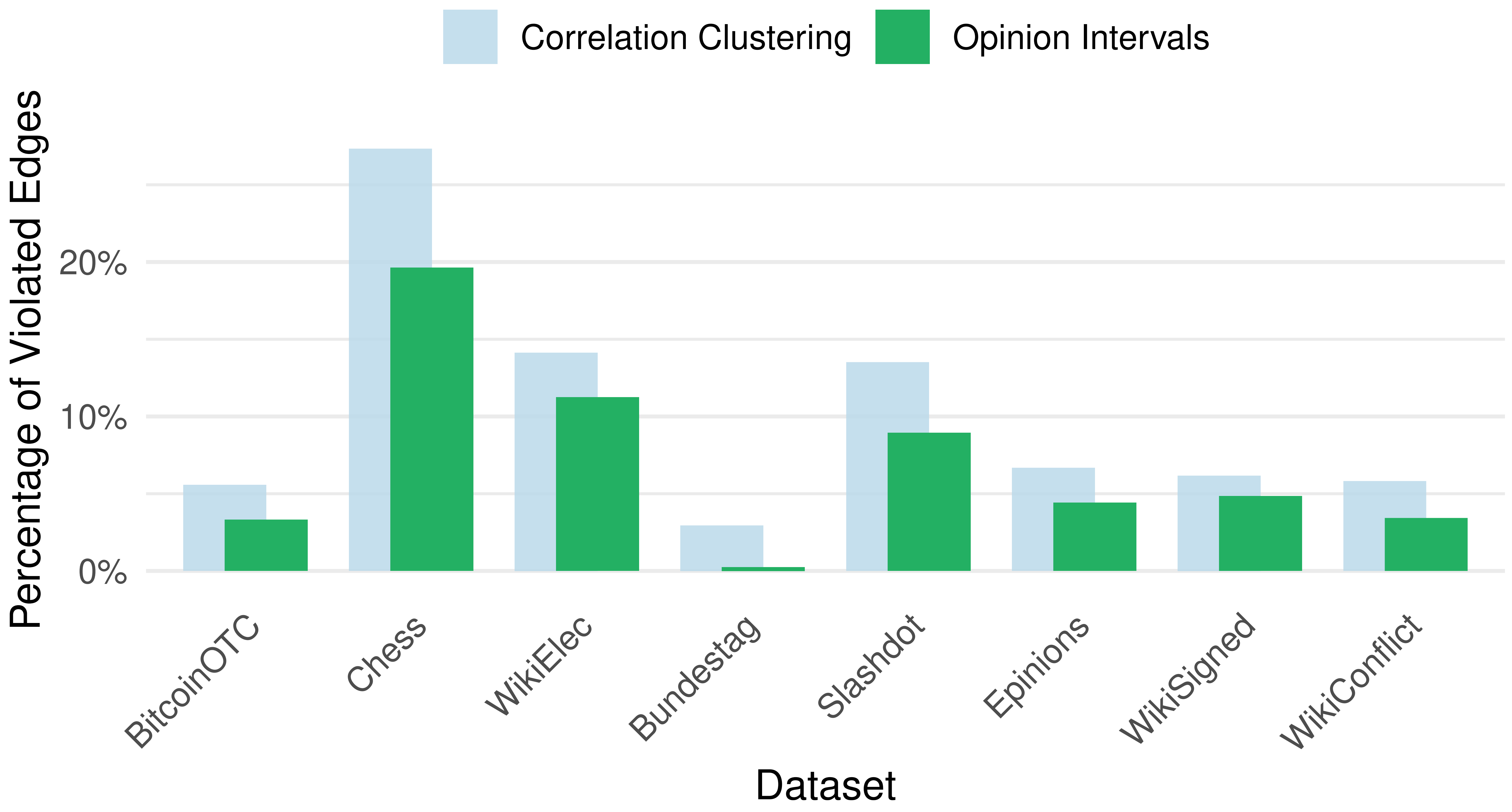
   Reuse solution on $V \setminus V_i$

   Assign V_i greedily
3. Combine solutions across all V_i



Experiments

38% fewer disagreements than Correlation Clustering



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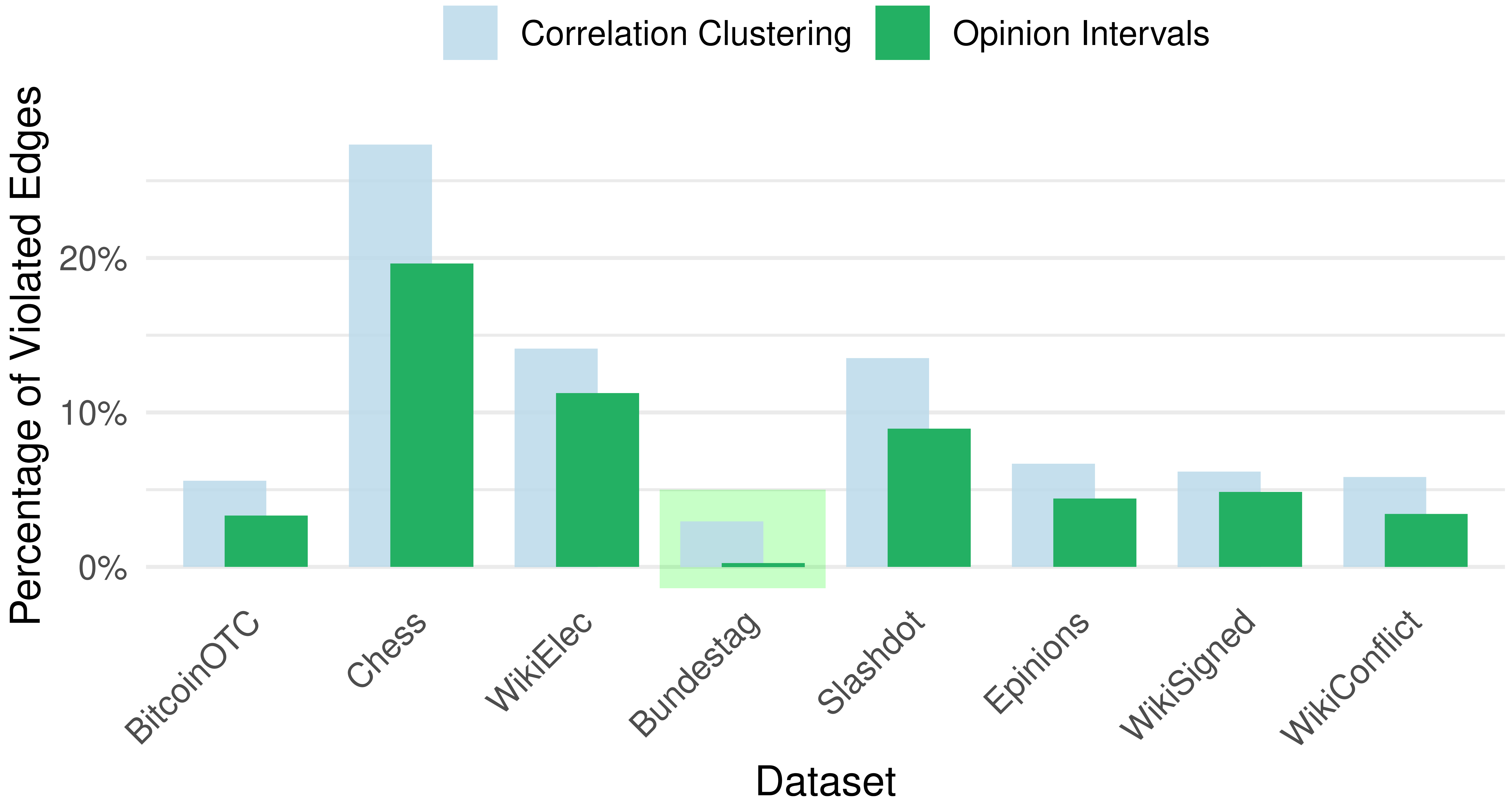
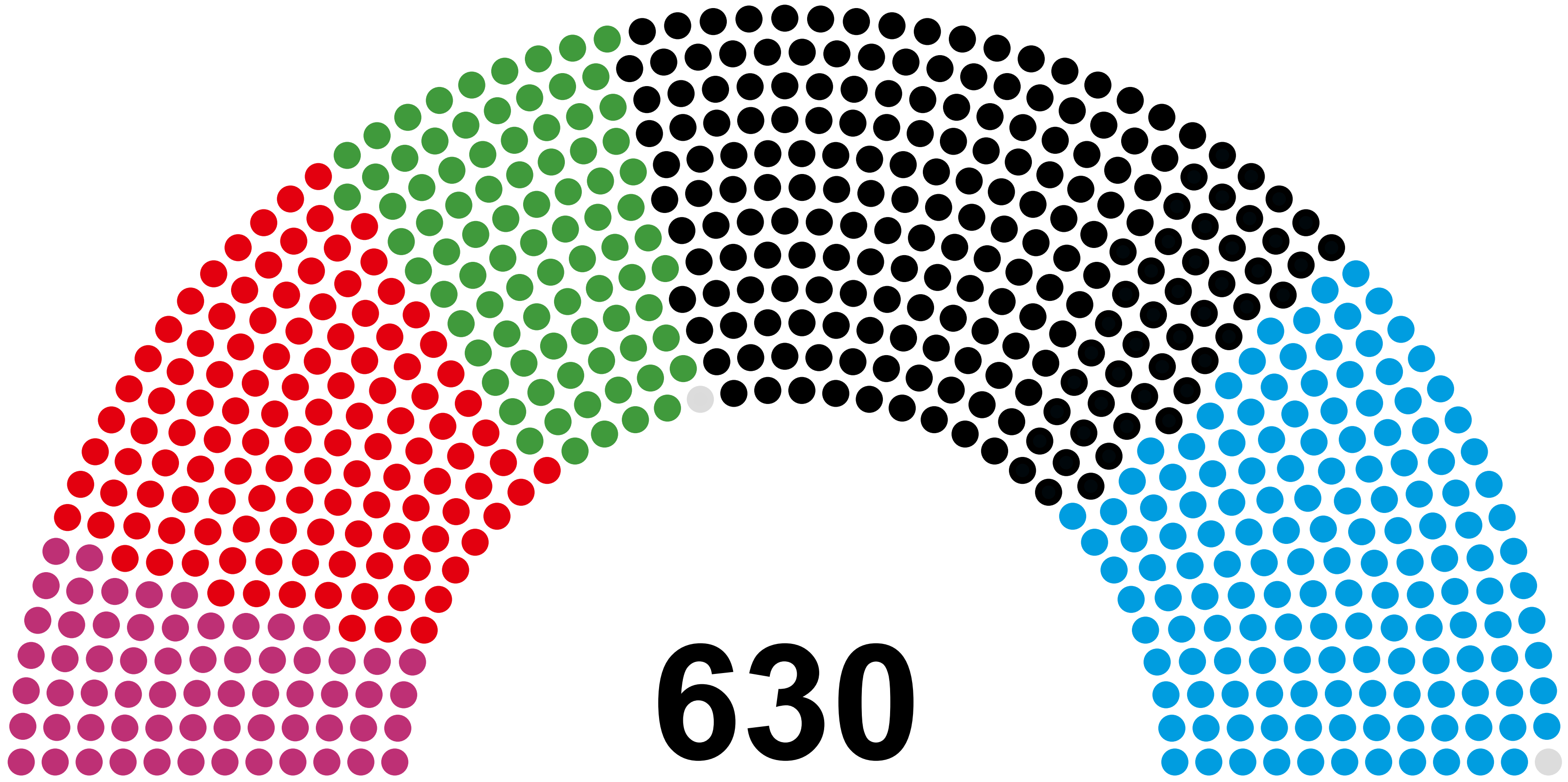
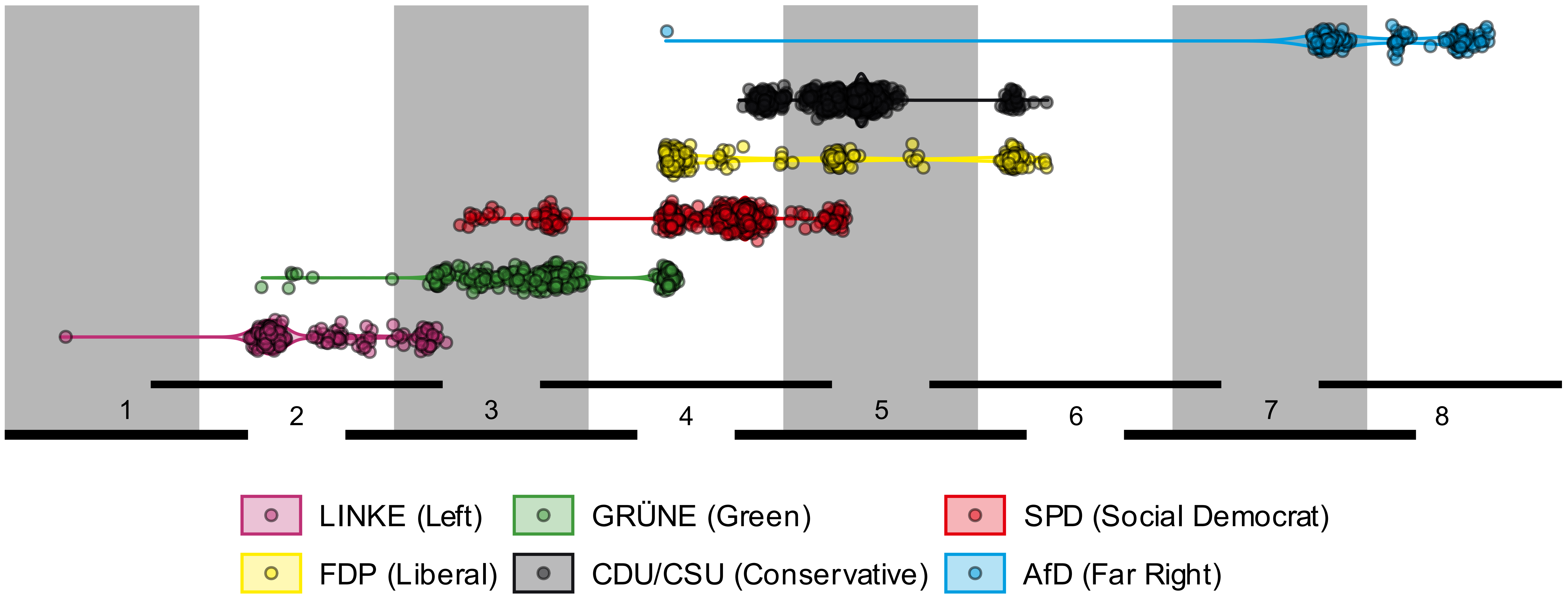


German Bundestag

- Multi-party political system
- Coalitions are formed to achieve majority
- We create a signed graph based on historic co-voting behaviour



Reconstructing the German political spectrum



Conclusion

We introduced the problem **Best Interval Approximation**

We provided a **PTAS** for complete graphs and a fixed interval configuration

We provided scalable and effective **heuristic algorithms**

Open Questions

Can we find better **approximation guarantees** in the unrestricted setting? (>75%?)

How can we efficiently find interval structures for **Best Interval Approximation?**

Can this approach be extended to **higher dimensions?**